DOCUMENT RESUME

ED 077 734 SE 016 292

AUTHOR	Kieren, Thomas E.
TITLE	Research on Computers in Mathematics Education, IV. The Use of Computers in Mathematics Education Resource Series.
INSTITUTION	ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, Columbus, Ohio.
PUB DATE	Apr 73
NOTE	43p.
AVAILABLE FROM	Ohio State University, Center for Science and Mathematics Education, 244 Arps Hall, Columbus, Ohio 43210 ($1.00)
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	Computer Assisted Instruction; Computer Oriented Programs; *Computers; *Curriculum; *Instruction; *Mathematics Education; Problem Solving; *Research Reviews (Publications)

ABSTRACT

This last paper in a set of four reviews research on a wide variety of computer applications in the mathematics classroom. It covers computer-based instruction, especially drill-and-practice and tutorial modes; computer-managed instruction; and computer-augmented problem-solving. Analytical comments on the findings and status of the research are included. For the other documents in the series, see SE 016 289 through SE 016 291. (Author/DT)

ED 077734

SMEAC/SCIENCE, MATHEMATICS, AND ENVIRONMENTAL EDUCATION INFORMATION ANALYSIS CENTER

. . . an information center to organize and disseminate information and materials on science, mathematics, and environmental education to teachers, administrators, supervisors, researchers, and the public. A joint project of The Ohio State University and the Educational Resources Information Center of USOE.

SE 016 292

ED 077734

THE USE OF COMPUTERS IN MATHEMATICS EDUCATION
RESOURCE SERIES

RESEARCH ON COMPUTERS IN MATHEMATICS EDUCATION

by

Thomas E. Kieren
University of Alberta

ERIC Information Analysis Center for
Science, Mathematics and Environmental Education
The Ohio State University
Columbus, Ohio 43210

April 1973

ACKNOWLEDGEMENT

The author would like to thank the Division of Educational Research Services, University of Alberta, for the use of their Technical Documentation Collection in the preparation of this review.

Preface

Since computers were introduced into schools little more than a decade ago, they have been used with increasing frequency in mathematics instruction. Their usefulness as a calculation aid or as a problem-solving tool was an obvious incentive for application in the mathematics classroom. Curriculum developers began exploring the potential for having the computer present mathematics lessons, while others developed drill-and-practice materials. Some educators saw the feasibility of using the decision-making capabilities and storage capacity of the computer for managing instruction.

In this paper, Dr. Kieren reviews research on a wide variety of computer applications. He brings to the task personal experience with most of these applications, as a teacher and as a researcher. The review not only summarizes and synthesizes the research, but also presents analytical comments on the findings and the status of the research. It should be helpful to educators at all levels who want to know, "What does research say about the use of computers in mathematics instruction?"

Marilyn N. Suydam
Editor

This publication was prepared pursuant to a contract with the Office of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official Office of Education position or policy.

THE USE OF COMPUTERS IN MATHEMATICS EDUCATION

RESOURCE SERIES

This is a set of papers and bibliographies addressed to both mathematics teachers and mathematics educators. An introductory paper discusses the general role of the computer in education. A second paper considers the use of computers in what is at present their most widely-used role, as a tool in mathematics problem-solving. A third paper reviews research related to computer uses in mathematics education. A three-part bibliography includes selected references on the general role of computers, on language and programming, and on mathematics instructional applications.

The titles in this resource series are:

The Use of Computers in Mathematics Education:
I. COMPUTER INNOVATIONS IN EDUCATION by Andrew R. Molnar

The Use of Computers in Mathematics Education:
II. COMPUTER-EXTENDED PROBLEM SOLVING AND ENQUIRY by Larry L. Hatfield

The Use of Computers in Mathematics Education:
III. BIBLIOGRAPHY
- Part 1. General Educational Role
- Part 2. Languages and Programming
- Part 3. Mathematics Instruction Applications
  - A. Teaching About Computers
  - B. General Uses
  - C. Tutorial and Practice Modes
  - D. Problem-Solving Mode

The Use of Computers in Mathematics Education:
IV. RESEARCH ON COMPUTERS IN MATHEMATICS EDUCATION by Thomas E. Kieren

The ERIC Information Analysis Center for Science, Mathematics and Environmental Education is pleased to make these papers and bibliography available.

Jon L. Higgins
Associate Director for
Mathematics Education

Research on Computers in Mathematics Education

A profound problem facing mathematics educators today is that of reconciling the human and the technological aspects of instruction. Certainly computer applications are questioned in this light. Can the computer contribute in other than trivial ways to instruction? If so, does it do so more effectively than do less-mechanized methods? Does computer-aided instruction imply a uniform curriculum for all and a high degree of imposition from "above"? Can the computer be used effectively to implement curriculum and instruction related to the human concerns in mathematics instruction?

The research on a wide variety of computer applications reviewed in this report does not in general address itself to any but the second question directly. Perhaps because of limited resources much of the research cited has a "product research" orientation. That is, the researcher involved frequently needed to produce a usable product and the research was then done on this product, whose necessarily hasty construction left it less than optimal with respect to the aspects being researched. Further, some of the research was done in some readily available "test market" of intact classes in less-than-controlled settings. Because of the nature of the computer and the resources needed to develop adequate software, some of the results exemplify Oettinger's (1969) criticism. He suggests that positive results on small-scale studies dissolve when the study is implemented on a large enough scale to be practical.

These general criticisms not withstanding, the research on computer applications does yield interesting information and suggest useful trends for future research. The review that follows will center on three very broad areas of application: computer-based instruction, computer-managed instruction, and computer-augmented problem solving. In addition, the first of the above will deal with research done using the computer as a research medium, as illustrated by the work of Suppes (1967), Jerman and Rees (1972), and Heimer and Lottes (1973).

Computer-Based Instruction

For the purpose of this paper, computer-based instruction (CBI) will be defined as that in which the student actually interacts with a computer which has been pre-programmed to provide this instruction. Included in this definition are the familiar drill-and-practice and tutorial CAI modes of instruciton.

Drill-and-Practice

Under computer-based drill-and-practice procedures, a child interacts with a computer via one of various types of computer terminals or via a touch-telephone. The computer, on a daily basis, provides 3 to 10 minutes of directed and corrected drill on problems which its program determines that the student needs. This determination is based on the student's history and his most immediate responses as well as on student desires in some cases.

Much of the work in this field is based directly or indirectly on the work done at Stanford (Suppes, 1964; Suppes and Morningstar, 1969; Suppes, Jerman and Dow, 1968; Suppes, Jerman and Groen, 1966). Parkus (1970) reports that there were some 470 terminals and 16,000 students in the United States using drill-and-practice CBI in 1969. The popular press has been full of rather glowing reports with respect to computer-based drill-and-practice. The research results are also rather positive. In a California study involving six grade-levels in seven schools, students whose arithmetic instruction was supplemented with drill-and-practice CBI programs had significantly greater pre-post gains on the Computation section of the Stanford Achievement Test (SAT) at grades 2 and 3, on the Concepts section at grade 3, and on the Applications section at grade 6, than similar cohort* groups having no computer-based drill (Parkus, 1970). In a no-control-group study in Kentucky, Parkus (1970) reported that after an average of 3.3 hourr of drill-and-practice, junior high school students gained over one year in computation level and two full years in concepts on the SAT. In a study done in Mississippi (Parkus, 1970), differences similar to the California study are cited, with significant differences favoring CBI drill recorded at all grade levels 1-6 in Computation, at grades 3 and 6 in Concepts, and at grade 6 in Applications. Carruth (1971) suggests, with respect to the same Mississippi group, that CBI drill was ineffective for those working at the lowest levels of the program. In a field study (S.R.A., 1970) of a program of 400

---

* "Cohort" groups are groups considered "comparable" to the experimental groups in a study.

hierarchical skills involving 292 grade 6 children, students using CBI drill-and-practice up to 20 minutes per day and from 1 to 16 hours total showed significant gains on a Facts Test and a Computation with Whole Numbers Test when compared with a control group. There was no difference in Total Computation scores (which presumably included content on fractions, etc.). Davies (1972), in a study of drill based on the California curriculum, found that low ability students using the computer gained 2.5 months more than a cohort group not using a computer with a year's work. Cole (1971) did a study with two experimental and eight cohort classes of grade 9 students. He found that the experimental students scored significantly higher than control students on computation with whole numbers and fractions.

There are results which run counter to the rather favorable ones cited above. Crawford (1970) found that students receiving 3 to 15 minutes of drill per day did no better on a computation test than a cohort group receiving no extra practice. Shaw (1968) reported a similar result. That a computer-based drill-and-practice program must be run extensively to be effective is suggested by Abramson and Weiner (1971). They note a reduced effect of the computer-based drill-and-practice program in the New York City schools the second year (1969-70) of operation. This they attribute to the fact that student exposure was only one-third that planned, resulting in students receiving no more effective drill than was traditionally done.

Aside from achievement are there any other effects of computer-based drill-and-practice on children? Once again the popular press reports claim that children like the computer or relate to it better than a human teacher. On the other side editorialists suggest dehumanization. The actual research on this aspect and on just what features of drill-and-practice programs enhance effectiveness is limited. Brod (1972), in a study of 16 students previously and currently (at the time of the study) using CBI drill and 34 students just beginning such a program, found that students did respect the computer as a teacher. These students--especially those having their first experience with computers--having formed authority relationships for goal attainment with the computer, had a reduced perception of a teacher's task-specific authority. Both the S.R.A. study (1970) and Cole (1971) report positive attitude changes toward mathematics with the use of CAI, but Cole found no change in attitude to school and no change in student attendance patterns.

In a study with 55 grade 5 students, Whitcomb (1972) found that although a CAI group learned more than a cohort group, there was no difference in learning that could be attributable to reinforcement given either on a fixed ratio or variable ratio basis. Neither was there a difference if this reinforcement was of high or low intensity. Barnes (1971) in a small study found no differences in achievement between groups of fourth through eighth grade students who chose their 30 problem types and groups whose 30 problems were selected by the computer. Achievement did not appear to be significantly affected by whether the student had a choice of feedback style or not, nor by what this style was. Schoen (1971) found that feedback which told a student why he was incorrect significantly increased achievement compared to feedback which just indicated incorrectness. In this study involving 60 randomly assigned pre-calculus university students, Schoen also found that personalizing feedback by including the student's name resulted in a significantly better attitude to the instruction than that generated by non personalized feedback.

While the question of dehumanization was not clarified in the research, there is evidence that students like CBI drill-and-practice. This effect is heightened by personalizing results, but the contribution of student choice and of reinforcement levels remains to be substantiated.

Drill-and-Practice Related Activities

In almost every computer-based drill-and-practice program, a student upon mastery of a particular level of exercise receives more-complex or difficult exercises. Similarly upon failure at a particular level a student will receive a less-difficult exercise set. The interesting question of what constitutes a more-difficult or a less-difficult exercise has been the focus of several studies. Suppes (1967) reported work on process models in arithmetic. With fourth-grade addition of whole number problems, linear regression models were used to predict difficulty of items in terms of logarithmically transformed proportions and response latencies. Using 24 fourth-grade students, with 38 exercises as data points, the Suppes group attempted prediction using three process variables: the number of steps, the magnitude of the sum, and the magnitude of the smaller summand. Regression equations resulted in correlations of .86 both between predicted and observed proportions and between predicted and observed [illegible]. The number-of-steps variable was the most potent. In a similar study with verbal problems Suppes, Loftus, and Jerman (19[illegible]) found that using six variables in a regression equation [illegible] for nearly 50 per cent of the variability in [illegible], in a study with 27 grade [illegible] students and some 68 word problems administered by the computer. It is interesting to note that in this study the students only sequenced the computations which were processed by the computer. The goodness-of-fit of predicted-to-observed data was not good: although for easy problems the fit seemed good, this decreased markedly with more difficult items. The prominent variables in the equation were the sequential variable (was the problem like the one before it), conversion (was conversion of units required), and operations (how many different operations occurred).

In perhaps the most extensive study to date, Jerman and Rees (1972) studied both the computer data generated in the above study and hand-calculated problems. They used primarily the same methodology as above with a greatly expanded set of variables. With variables such as memory (a linear combination involving formula, numerals and different operations), S_7 (number of displacements in order of operation from the just-previous problem), and complex order variables, the regression model accounted for 82 per cent of the variance in the proportion correct of 30 problems. Adding some hand-calculation variables, the model accounted for 87 per cent of the variance with cognitive factors such as memory and sequence giving way in importance to the hand-calculation variables on hand- [illegible] tea [illegible].

W[illegible] respect to the question [illegible] difficulty, this latter study differs from the previous [illegible] without concern for the stability difficulties [illegible] predicting 30 item difficulties using 19 variables [illegible] linear combinations of one another. The Jerman and [illegible] present other features. To their credit, the variables used are complicated, perhaps reflecting the really [illegible] nature of something as apparently straight-forward as the typical school arithmetic "word problem." Yet the very complexity of the variables reduces their usefulness in generating problems [illegible] particular difficulty level. The above studies suggest [illegible] to grips with what is meant by a priori difficulty of standard elementary arithmetic exercises. They also [illegible] actual practices, for computer generation [illegible] based upon models of item difficulty.

Tutorial Computer-Based Instruction

In this paper tutorial computer-based instruction is meant to imply that the student receives instruction on new material directly from the computer. The computer also monitors student learning activities with respect to interaction [illegible] the system. Although varied in content, it is probably safe to say that most drill-and-practice programs have basic similarities. Such is not the case with tutorial programs. Many reported below were administered via typewriter terminals; others were based on sophisticated systems with cathode ray tubes, rear-screen projectors, and audio aids. The programs as well vary greatly. One thing is certain: computer-based tutorial instruction is to date expensive (Lynch, 1971). Even using CBI in relatively simple ways and with some 4000 students, Montgomery County, Maryland found the cost in 1970/71 to be 94.98 dollars per user-hour including training and development costs (Dunn and Wastler, 1972). Perhaps it is the cost factor and a related time factor which limits the extent of the studies cited below

There have been a number of studies at several grade levels which have noted a positive effect favoring tutorial CBI over conventional instructional methods. Schurdak (1967) reports a CBI group superior to both programmed instruction and traditional instruction in terms of students learning FORTRAN. In a study of 80 calculus students randomly assigned to traditional or CBI treatments, Ibrahim (1970) found the CBI group significantly better than the traditional groups in terms of immediate test scores; there were no differences on a retention test. Dunn and Wastler (1972) found that both CBI and traditional grade 11 classes gained from pre- to post-test on the Blyth Second Year Algebra exam; however, the CBI group gained significantly more. Ostheller (1971) found CBI significantly better than programmed instruction or traditional instruction in probability and statistics in a study involving 38 randomly-assigned students. He found students preferred teacher-student interaction, however. Isaac (1972) used linear and branched CBI programs to provide instruction in logarithms. He assigned a stratified random sample of 45 students to linear, branched, or classroom instruction on a random basis. After ascertaining that all students actually used branches in the branched program, Isaac found that the branched program was superior to the other two instructional modes for all ability levels and for factual, conceptual, and problem test items after two weeks of instruction. This difference was dramatically pronounced for lower-ability students, with the branched program proving greatly superior to the linear program which in turn was better than the classroom instruction. The linear-program superiority did not stand up for other ability groups. Like Ostheller (1971), Isaac found no differences in student attitude toward mathematics between CBI and classroom students. In fact it appears that favorable CBI results are not related to enhanced attitudes on the part of the students.

Not all CBI research results are positive. Confer (1971), in studying the General Mathematics aspect of the Pennsylvania Consortium project, used randomly-assigned repeating summer-session students. There were no differences in learning, as demonstrated on the SAT Form 4, between CBI and traditional groups. He also found no greater attendance in the CBI group. Confer also reported that students who used the terminals for extended periods of time per day suffered from fatigue and frustration. Kanes (1971) found no differences among a guided-discovery CBI, an expository CBI, and a cohort group on a post-test or on a retention test. Riedesel and Suydam (1967) found no differences between CBI and teacher-taught groups of preservice elementary teachers in terms of content learning. Ward and Ballew (1972), in two studies of elementary education majors learning set theory, noted no differences between computer and control groups. They used two types of terminals, one with computer-controlled video aid and one without computer-controlled video aid (this group had a book of visuals). Of the three randomly-assigned groups the non-visual-terminal CBI group fared best, although not significantly better than the visual-terminal group or the control group. The authors concluded that the visual-terminal expense was not justified.

There were a number of studies which looked at program-design factors or use factors of tutorial CBI instruction. In contrast to Isaac (1972) above, Melaragno (1967) in a study partly involving CBI found no differences among groups using linear or branched programs. Gay (1971) studied placement of reviews in CBI. She found in a study of 53 grade 8 students that reviews one day, one week, and two weeks after instruction were equally effective, and better than no review, in terms of achievement. She also studied 67 grade 8 students receiving two reviews. In this study she found that groups with reviews after one and seven days were superior to groups with early (one and two days) or later (six and seven days) reviews. She also found that the number of questions needed to reach criteria was cut by a factor of two from the first review to the second. The latency on the first review was one-half the latency on original mastery, while in the second review latency was cut by 75 per cent of the original. Klement (1971) studied feedback in a five-lesson program for 48 under-educated adults. He found no difference in groups receiving knowledge of results only, reinforcement, or reinforcement correlated to the level of the response. O'Neil (1970) induced stress by giving negative feedback on errors. Using female first-year university mathematics students as subjects, he found that high-anxious students were significantly better than low-anxious students in stress-feedback situations. These student groups were not different in non-stress settings. Overall, low-anxious students did better than high anxious students on easy material and generally made fewer errors. Igo (1972) used a game of Battleship to determine high-and low-risk students. He then assigned students to deductive and inductive programs for instruction on a mathematical task. The risk-taking level had no effect, as deductively-taught students made fewer responses and took less time to reach criterion than did inductively-taught students. Bissent (1971) found that students were good judges of needs in selecting program modules; however, he felt that author-definition might be better for determining uniform subject matter. The Ward and Ballew (1972) study mentioned above pointed to reduced cost through use of less-sophisticated equipment in CBI. Love (1970) found no difference in the learning of Abstract Algebra between students working alone or in pairs at the terminal. Cartright (1973), in a non-mathematical study, found this result holding for groups of up to four persons, at least for a three-lesson program.

One finds it nearly impossible to summarize the results noted above. There are several studies mostly at higher grade-levels and with university students supporting the use of tutorial CBI instruction. This support is by no means uniform. There appear to be ways, such as timing of reviews, use of branching, use of deductive teaching, and group use of terminals, to increase instructional efficiency under CBI.

This summary along with the work of Chapman (1970) and Luskin (1971) suggest that skilled individuals are a major need in the development of tutorial CBI instruction. The work of Hicks and Hunka (1972) suggest that these skills are such that teachers can learn them. It may be that the limited use of tutorial CBI is associated with a very low level of teacher education in the needed skills.

Tutorial Computer-Based Instruction Related Activities

It is clear that any instructional research would have some implications for tutorial CBI instruction. It is beyond the scope of this paper to engage in a review of such research. However, one project--the Paradigms Project at The Pennsylvania State University--uses tutorial CBI as a research tool and could contribute to more efficient, theory-based tutorial CBI instruction. This project is based upon the assertion that instructional research should be modeled on the characteristics of empirical science (Heimer and Lottes, 1973). Using an input-output grid, components of which arise from the work of Bruner (1966), instructional theorems are generated. Highly replicable computer-based empirical tests are then done. For example, Klein (1970) found that if the input mode of objective A (of like content to objective B) is the output mode of objective B and the input mode of B is the output mode of A, then explicit instruction on either objective leads to mastery of the other without instruction. Although the percentage of such generated theorems supported empirically was not high (Klein, 1973; Hirschbuhl, 1973; Farris, 1973), the direction of this research may prove fruitful in a later theory applicable to the instructional engineering of tutorial CBI.

In terms of learning problem-solving algorithms, Hostetler (1973) found, contrary to theory, that students preferred an algorithm learned second to that learned first. Knowledge of the scope of applicability does not appear to affect grade 5 students' judgment in algorithm-choice.

Using a two-dimensional grid relating congruent triangles according to the necessary transformation of mapping one onto the other and according to relative positions, Paquette (1973) generated some 30 instructional hypotheses similar in structure to those noted above. He used samples of accelerated and regular students studying geometry (grades 9 and 10) in a computer-based instructional environment. Paquette found substantial support across both samples for hypotheses which suggested that particular objectives would be mastered without instruction, given instruction-to-mastery on others. It was found that instruction on congruency where the pairs were related by a translation and had no overlap was especially generative of mastery congruency objectives using other transformational and relative positional combinations. Paquette found that the non-accelerated students could achieve mastery of objectives where congruency depended upon a reflection and where the triangles were non-overlapping but had an infinite point-set intersection only after explicit instruction. Bowers (1973) in a related study considered effects of instruction on objectives using the non-intersecting translation alone and that using instruction-to-mastery of objectives involving reflections and rotations as well. The first treatment was considered single-configurational while the second treatment was multi-configurational. The criterion class of objectives was a set of five classes involving other transformation-position pairs. Bowers used the same sample as Paquette. Although there was considerable transfer from both single and multi-configurational instruction there was little support for the hypothesis that all objectives in the criterion class would be mastered without explicit instruction.

Sawada (1973) and Hopkins (1973) studied the problems of students traversing learning hierarchies. They generated hypotheses similar to those of Klein (1973), but with special reference to Piagetian-derived notions of reversibility and transitivity. Using 30 randomly-selected grade 5 students in computer-based instruction on fractions, Sawada and Hopkins found that explicit instruction was necessary for mastery of objectives derived from mastered ones using principles of reversibility and transitivity.

While none of The Pennsylvania State University studies have been replicated, their results and procedures can give some guidance for instructional design for CBI. In total and individually they have demonstrated novel ways of generating instructional hierarchies and hence instructional tasks. With the exception of the Paquette study, they all suggest that explicit instruction is most frequently necessary to insure mastery of objectives related to previously-mastered ones in a wide variety of ways.

Computer-Managed Instruction

For the purposes of this paper, computer-managed instruction (CMI) will be taken to mean a total system of curriculums and related evaluation instruments, student records prior to and within interaction with the system, and a means for appropriately assigning to each student the best curriculum on the basis of his history and in light of available resources. Although CMI could subsume CBI in most current applications, it is not used in this fashion. Rather the computer is used as a monitor and as an information system for designing instruction generally produced in other ways.

The actual research efforts with respect to CMI are limited in number. In this paper such research will be taken to include computer-assisted testing and computer-managed reporting, although these are only a part of CMI.

One important instructional management concern, particularly in individualized instruction, is the initial placement of students in the instructional hierarchy. Typically this is done either by fiat or by an entrance testing program. While the latter is desirable, if the program has numerous objectives in sequence, the testing program may be unbearably long. Ferguson (1970) developed a computerized testing procedure related to IPI.* Under his program a student would receive generated items for a middle-range objective until he scored above a set criterion or below a set minimal level on four or more items. Such mastery or failure would cause a student to move "up" or "down" in the hierarchy. Such testing would continue until the student reached a stable level. Like other schemes of the tailored-testing variety, Ferguson found that such tests provided the same information as conventional tests but in less than half the time and items. The scheme proved to

*IPI is the acronym for Individually Prescribed Instruction, a program developed by the Learning Research and Development Center at the University of Pittsburgh.

have great reliability in aiding instructional-placement decision-making.

Clover (1972) and Lee (1972) studied computer-aided progress-reporting. In the Clover study with grade 4 pupils, a list of 99 behavioral objectives was developed. A mechanism for using optically-screened teacher-report sheets was created, and teacher reports were thus stored and transferred by computer to pupil reports at appropriate times. Teachers in the study tended to use this reporting scheme either consistently or a few times, then dropped it. The teachers' level of use correlated highly with their opinion of the system. Lee (1972) studied parent reaction to a similar reporting system. Parents and pupils reacted favorably to the five reports developed under the system, although some parents preferred parent-teacher conferences.

Considering that such systems as PLAN (Flanagan, 1972) have been operating for four years, there has been only limited investigation on total management systems. Kriewall (1970) analyzed the development of the program for self-paced, self-selected learning. He proposed a systems approach, sponsoring a computer-managed instructional system based on a value-testing scheme as a necessary condition for developing such learning approaches. Westrom and Zarsky developed a set of computer programs to allow for the management of instruction called Teacher Authored Instruction Manager (TAIM) (Westrom, 1972). This system enables teachers to function either as users or authors in designing the logic, displays, and tests in the system. TAIM is designed to produce a lesson or first-level student plan for a day. It scores student tests daily on a flexible basis and produces directions for the next possible learning experience based on student records. The teacher can obtain records of student learning-paths, system bottle-necks, and numerous other statistics, as well as send personal messages via the system. The teacher also acts as author in designing and redesigning actual curriculum. A feasibility study has shown that teachers are capable of designing curriculum with TAIM, and of modifying this curriculum to match the needs of students.

Computer-Augmented Problem Solving

The ruberic "computer-augmented problem solving" covers a wider area of application than the name would suggest. For this paper it is taken to mean any situation in which the student from elementary school through university learns to program a computer and uses this tool to learn mathematics. Many persons have claimed that programming a computer significantly aids learning of mathematics and enhances learning skills (Hatfield and Kieren, 1972; D.C. Johnson, 1966; Haven, 1970). Perhaps the boldest claim is made by Papert and Soloman (1972) who suggest that the computer is the medium for learning how to learn. (In their work, elementary students program various devices such as a cybernetic turtle, which does such things as "Turtle Geometry.")

Computer-augmented problem solving has enjoyed rather widespread application. Since there is a wide variety of means of implementing this mode of instruction--remote terminal, mini-computers or by carrying programs to school computer-centers--many schools have made such a program available to at least some students.

One might expect that this relatively widespread use might be accompanied by a rather large body of research. Such is the case if one considers reports of use-attempts (e.g., Smith, 1971). However, more carefully designed studies, either formative or summative, are not as numerous. Included in the research reported below are studies concerned with computer language learning and means of processing programs, as well as those studies concerned with the actual effects of student-use of the computer in terms of mathematics learning.

From the considerable number of students using computer-augmented problem solving, one might conclude that programming aptitude and skill can be developed. However, it has been reported in Kieren (1968) that although all students in his two-year study achieved passable skills, there was a range in aptitude, skill, and interest in programming. Results of King (1972) and Hatfield (1969) with lower achievers also suggest that programming aptitude and skill is an issue. Carol Ann Alspaugh (1972) focused on predicting programming aptitude. Although her university-sample exam results cannot be generalized to high school students, her results are interesting as she attempted to correlate various factors with BAL and FORTRAN language-learning achievement. Although there were minor differences between the regression equations using the different languages as criteria, it was found that mathematics background in terms of high school and university courses was the major predictor, with personality factors making other strong contributions to the equations, which had a multiple-correlation of .62. Personality adjectives related to good programming appeared to be non-social, reflective, and non-vigorous. Perhaps because it was subsumed by the background variable, a programmer-aptitude test from IBM made only a small contribution. John Alspaugh (1971) compared 14 high school juniors and seniors with 23 college juniors and seniors of similar general ability. He found that, although the groups differed significantly on the IBM Programmer Aptitude Test, doubling the high school students' instruction time to two hours per week for a semester resulted in no difference between groups in their demonstrated learning of FORTRAN.

Little actual research has been done comparing languages used in computer-augmented problem solving. Dennis (1972), in studying how students used languages in solving problems, suggested that a multi-level language was needed. He also suggested that an instructional format encouraging blocking of instructions was useful. Feurzig (1969) suggested that the language LOGO promoted self-conscious literacy about problem solving and suggested programming as a means of clear, precise expression of mathematical thinking and skills. In a study with university students, Knodel (1972) studied BAL and FORTRAN learning. He found no order effect in the learning of the two languages, and also reported that BAL knowledge deteriorated more rapidly than FORTRAN (which is not surprising considering the amount of detail in the former). Students seemed partial to the first-learned language, but much more partial to the language related to their vocational choice.

In terms of processing, neither Skelton (1972) or Pack (1971) found learning differences between groups accessing the computer via time-sharing and groups accessing the computer via batch-processing (from 1 to 24 hour return time). Pack found that students preferred time-sharing, while Skelton observed that time-sharing students submitted significantly many more program runs per week. He also found time-sharing to be twice as costly.

Studies into the effectiveness of computer-augmented instruction have been done using a variety of mathematical topics and at several grade levels. Bitter (1971) studied the effects of using the computer to learn calculus, at three colleges each having an experimental and a control class. Students in the experimental setting wrote BASIC programs to solve special homework problems. During the semester of computer-use, the computer groups scored significantly higher than the control groups on the COOP Calculus Tests. No differences were noted in a follow-up semester with no computer application. Holoien (1971) found, in a study of four classes randomly assigned to computer and non-computer treatments, that lower ability students particularly were aided by the computer treatment. The computer treatment involving 50 per cent of the classes and 50 per cent of the assignments appeared especially effective with limit and function concepts. Bell (1970) found similar differences favoring a similar computer treatment on understandings in calculus, but not on techniques of calculus. Schmidt (1970), however, in a study of 30 junior college students, found no differences between computer and non-computer students in introductory calculus achievement. In this study the computer was used as a calculator.

Hatfield and Kieren (1972) studied the effects of computer-use for two years at grades 7 and 11. Students were randomly assigned to computer or non-computer treatments and blocked for analysis purposes on previous mathematics achievement. At each grade-level extensive lists of programming settings were developed involving skills, concepts, and problem solving. During Year 1 of the experiment at the grade 7 level, students in the non-computer class scored significantly higher than those in the computer class on the unit test on numeration, a unit during which the computer classed initially-learned programming. This effect did not appear in Year 2 of the experiment. Particularly in Year 2 the computer group in grade 7 achieved at a higher level than the non-computer group. This difference was significant on the unit test on number theory, the Contemporary Mathematics Test, and the Thought Problems Test, the latter two being final examinations. In analyzing nearly 600 items at the grade 7 level it appeared that computer contributions mainly appeared in conceptual or problem-solving items. At the grade 11 level, the computer-group means were generally higher than those of the non-computer-group over the two years. In contrast to the grade 7 data showing that the lower-achieving students fared relatively less well under computer treatment than did high-achievers, there was a uniform trend on tests at the grade 11 level for a differential effect favoring average over high-achieving students in the computer classes. Significant differences favoring the computer class were observed on two exams while a significant difference favoring the non-computer class existed on a unit test on trigonometry. A study of over 300 items suggested that the computer made its greatest contributions on organizational and complex-skill items.

Ronan (1971) found similar results with high school students. The computer group in his study was favored on exponential and logarithmic functions, mathematical skills, and logic and reasoning. The control group was favored on trigonometric identities and formulae. These findings closely parallel the grade 11 Hatfield and Kieren (1972) results. Hoffman (1971) found no significant differences between computer and non-computer groups where the computer group programmed using BASIC. The computer appeared favored on generalizing items related to the notion of debugging. Haven (1970), in a less formal study of high-school students, found that students who used flow-charting showed larger gains in abstract reasoning and scholastic aptitude than did a control group. A group which wrote and executed programs did much better still. Katz (1971) found that using class time to run programs was ineffective. In a study of nine randomly-assigned classes, the group which wrote but did not their programs was significantly favored on the COOP Algebra II test over a group which ran their own programs and a control group. King (1972) found that, for low achievers, a program of mastery learning and flow-charting was superior to a mastery-learning treatment, and to a treatment involving computer programming of problems in addition to flow-charting.

Washburn (1970) found programming using CUPL strengthened mathematical understanding and led to a more positive attitude towards mathematics at grades 7, 8, and 12, and with college freshmen. At all levels the high ability students seemed relatively favored by the computer treatment. Although clearly the treatments were not comparable, these results are in contrast at the upper grade levels to the findings of Holoien (1971) and Kieren (1968).

In another study with results which run contrary to the above trends, Johnson (1971) found that with grade 7 students in number theory, computer-programming did not improve achievement. This result appeared in a larger study of several topics and using other activities than computer-programming in other sections of the study.

Morgan (1969) with students in general education and Broussard, Fields, and Reussvig (1969) with low-achievers found that integrating mathematics with computing activities produced favorable results.

In summarizing the body of research discussed above, it would appear that though the evidence is not conclusive, the balance of the evidence is favorable with respect to computer-augmented problem solving. There do appear to be differences with respect to different topics. In addition, the interaction of various computer-augmented treatments with students of different aptitude-levels is apparent from this data and should serve as an interesting problem for further development and research.

In terms of physical operation it would appear that students do not need to run their own programs or access the computer directly to succeed. Although no conclusions on language-use can be gathered from this evidence, a preponderence of studies used BASIC. Other languages were used effectively, however.

There is some evidence in the aggregate of these studies that computer-programming does promote process skills such as organizing, generalizing, and problem solving. However, any definite conclusions in this area await further study.

Concluding Remarks

Can any kind of general conclusions be drawn from this body of research on computer applications in mathematics instruction? There is some support for such applications in all of the modes; this evidence is particularly strong in the areas of drill-and-practice and computer-augmented problem solving.

Supposing there exist favorable consequences from any or all such applications, is computer-aided instruction cost-effective? There is no answer to this question in this research. However, in all areas, using less-expensive operating procedures did not appear to adversely affect learning. If this were a firm conclusion, one might predict a big push to reduce costs through devising and adapting less-expensive procedures. Unfortunately, this effect may be an artifact of the short duration of most of the studies, the unsophisticated tasks addressed, or the less-than-optimal way in which the computer was used.

There is no evidence in the research cited in this review to suggest teacher-displacement by computers. Nonetheless, borrowing a notion from Walbesser (1972), it appears that the lower the puissance of the objective the more favorable the effect of computer-based application. If teachers put undue emphasis on computational efficiency in their objective-set, it appears that the related instructional tasks can be handled either by computer-based drill-and-practice programs or through the use of electronic calculators.

In the light of the current applications, is there an indication of research directions which will lead to applications having a more dramatic effect and thus making all computer applications cost-effective? It would appear that questions such as the following need be answered:

--If a computer is going to give or manage instruction, do programming models exist which allow management of sophisticated learning such as process or problem-solving learning?

--Can measures of process variables be designed to evaluate non-content effects of computer applications in mathematics? Can these be adapted to a variety of topics and at various levels of sophistication?

--Can process models of mathematical ideas be derived which describe the concept, skill, or process adequately, but which are simple enough to be routinized in a generative computer program?

--Can computing settings be invented which give a student the feeling of controlling a machine and simultaneously give the student access to his own thinking processes?

--Can control of the system, whether computer-based, computer-managed, or computer-augmented, in some important ways reside with the learner? If so, what are these important ways?

If the answer to such questions is "no," then the positive findings in the research cited in this review diminish in importance, and perhaps even a large reduction in cost would not result and probably should not result in anywhere near universal use of computers in mathematics instruction. Fortunately, though not yielding conclusive evidence, the research permits one to see that there is active work on underlying theory, on exciting management schemes, on more-complex instructional routines, and on new curricular and instructional settings for computer application. This activity suggests that questions such as those above can be answered in the affirmative.

REFERENCES

Abramson, Theodore and Weiner, Max. Evaluation of the New York City Computer Assisted Instruction Program in Elementary Arithmetic, Second Year, 1969-70. February 1971. ERIC: ED 047 962. 11 p.

Alspaugh, Carol Ann. Identification of Some Components of Computer Programming Aptitude. Journal for Research in Mathematics Education 3: 89-98; March 1972. ERIC: EJ 053 935.

Alspaugh, John W. The Relationship of Grade Placement to Programming Aptitude and FORTRAN Programming Achievement. Journal for Research in Mathematics Education 2: 44-48; January 1971. ERIC: EJ 032 680.

Barnes, Ospy Dennis. The Effect of Learner Controlled Computer Assisted Instruction on Performance in Multiplication Skills. (University of Southern California, 1970.) Dissertation Abstracts International 31A: 4538; March 1971.

Bell, Frederick Harold. A Study of the Effectiveness of a Computer-Oriented Approach to Calculus. (Cornell University, 1970.) Dissertation Abstracts International 31A: 1096; September 1970.

Bissent, W. Evaluation of a Modular Freshman Mathematics System with Student Control Options. In Minutes and Proceedings, ADIS, February 1971. pp. 21-22.

Bitter, Gary Glen. Effect of Computer Applications on Achievement in a College Introductory Calculus Course. (University of Denver, 1970.) Dissertation Abstracts International 31B: 6109; April 1971.

Bowers, Robert Garth. Toward a Theory of Sequencing: Study 1-4: The Effect of Triangle-Pair Configuration Variation on Achievement of Selected Classes of Instructional Objectives in Plane Geometry. Journal for Research in Mathematics Education 4: 110-113: March 1973.

Brod, Rodney Lynn. The Computer as an Authority Figure: Some Effects of CAI on Student Perception of Teacher Authority. (Stanford University, 1972.) Dissertation Abstracts International 33A: 139; July 1972.

Broussard, Vernon; Fields, Albert; and Reussvig, James M. A Comprehensive Mathematics Program. Audiovisual Instruction 14: 43-44, 46: February 1969. ERIC: EJ 001 360.

Bruner, Jerome. Towards a Theory of Instruction. Cambridge, Massachusetts: Harvard University Press, 1966. 171 p.

Carruth, Edwin Ronald. A Multiple Linear Regression Analysis of Computer-Assisted Elementary Arithmetic Achievement. (University of Southern Mississippi, 1970.) Dissertation Abstracts International 31A: 5691; May 1971.

Cartright, G. Social, Personality and Attitudinal Dimensions of Individual Learning with Computer Assisted Group Instruction. University of Alberta, 1973. In press. 215 p.

Chapman, Joseph Warren. A Determination of the Skills Required of Elementary and Secondary School Teachers in Schools Using Computer-Assisted Instruction. (The American University, 1970.) Dissertation Abstracts International 31A: 2785; December 1970.

Clover, Michael E. Study of the Feasibility of Computer Assisted Pupil Progress Reporting in 4th Grade Mathematics. (The University of Iowa, 1972.) Dissertation Abstracts International 33A: 1312-1313; October 1972.

Cole, William L. The Evaluation of a One-Semester Senior High School Mathematics Course Designed for Acquiring Basic Mathematical Skills Using Computer-Assisted Instruction. (Wayne State University, 1971.) Dissertation Abstracts International 32A: 2399; November 1971.

Confer, Ronald William. The Effect of One Style of Computer Assisted Instruction on the Achievement of Students Who Are Repeating General Mathematics. (University of Pittsburgh, 1971.) Dissertation Abstracts International 32A: 1741: October 1971.

Crawford, Alan N. A Pilot Study of Computer-Assisted Drill and Practice in Seventh Grade Remedial Mathematics. California Journal of Educational Research 21: 170-181; September 1970. ERIC: EJ 027 030.

Davies, Thomas Peter. An Evaluation of Computer-Assisted Instruction Using a Drill-and-Practice Program in Mathematics. (United States International University, 1972.) Dissertation Abstracts International 32B: 6970; June 1972.

Dennis, John David. Some Considerations in the Design of Computer Languages for Interactive Problem Solving. (The Ohio State University, 1971.) Dissertation Abstracts International 32B: 4611; January 1972.

Dunn, A. and Wastler, J. Computer Assisted Instruction Project. Final Report, Project Reflect. Rockville, Maryland: Montgomery County Public Schools, 1972. 502 p. ERIC: ED 066 876.

Farris, Dan C. Toward a Theory of Sequencing: Study 1-2: An Exploration of Selected Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation. Journal for Research in Mathematics Education 4: 104-105; March 1973.

Ferguson, Richard Leroy. The Development, Implementation, and Evaluation of a Computer-Assisted Branched Test for a Program of Individually Prescribed Instruction. (University of Pittsburgh, 1969.) Dissertation Abstracts International 30A: 3856; March 1970.

Feurzig, Wallace. Computer Systems for Teaching Complex Concepts. Contract No. NR-4340(00). Washington: Office of Naval Research. March 1969. Report No. 1742, Bolt, Barnaek and Newman, Inc.

Flanagan, John. The PLAN System as an Application of Educational Technology. Educational Technology 12: 17-19; September 1972.

Gay, Lorraine Rumbel. Temporal Position of Reviews and Its Effect on the Retention of Mathematical Rules. (The Florida State University, 1970.) Dissertation Abstracts International 32A: 237; July 1971.

Hatfield, Larry Lee. Computer-Assisted Mathematics: An Investigation of the Effectiveness of the Computer Used as a Tool to Learn Mathematics. University of Minnesota, 1969. 152 p. (See also Dissertation Abstracts International 30A: 4329-4330; April 1970.)

Hatfield, Larry L. and Kieren, Thomas E. Computer-Assisted Problem Solving in School Mathematics. Journal for Research in Mathematics Education 3: 99-112; March 1972. ERIC: EJ 055 232.

Haven, R. Laboratory Program for Computer Assisted Learning (Project LOCAL). In The Use of Computers in Secondary School Mathematics, Dudley Post, Editor. Newburyport, Massachusetts: ENTELEK, 1970. pp. 216-224.

Heimer, Ralph T. and Lottes, John. Toward a Theory of Sequencing: An Integrated Program of Research. Journal for Research in Mathematics Education 4: 85-93; March 1973.

Hicks, B. and Hunka, S. The Teacher and the Computer. Philadelphia: W. B. Saunders, 1972. 219 p.

Hirschbuhl, John J. Toward a Theory of Sequencing: Study 1-5: An Exploration of Selected Transitivity and Conjunctive Relationships Among the Enactive, Iconic, and Symbilic Modes of Representation. Journal for Research in Mathematics Education 4: 113-115; March 1973.

Hoffman, Irwin J. Effect of Computer Application on Generalization Skills and Achievement in a Second Year Algebra Course. (University of Denver, 1971.) Dissertation Abstracts International 32B: 2856-2857; November 1971.

Holoien, Martin Olaf. Calculus and Computing: A Comparative Study of the Effectiveness of Computer Programming as an Aid in Learning Selected Concepts in First-Year Calculus. (University of Minnesota, 1970.) Dissertation Abstracts International 31A: 4490: March 1971.

Hopkins, Layne V. Toward a Theory of Sequencing: Study 3-2: An Exploration of Transitivities Formulated from a Set of Piagetian-Derived Operations and Their Implications in Traversing Learning Hierarchies. Journal for Research in Mathematics Education 4: 121-123: March 1973

Hostetler, Robert P. Toward a Theory of Sequencing: Study 2-1: An Exploration of the Effect of Selected Sequence Variables on Student Choice in the Use of Algorithms. Journal for Research in Mathematics Education 4: 115-118: March 1973.

Ibrahim, Aziz Tawfik. A Computer-Assisted Instruction Program for Teaching the Concepts of Limits in Freshman Calculus (A Comparative Study). (State University of New York at Buffalo, 1970.) Dissertation Abstracts International 31A: 1689: October 1970.

Igo, Robert Vincent. Using Risk Taking Behavior to Assign Students to Instructional Treatments in a Computer-Assisted Instruction Program. (The Pennsylvania State University, 1971.) Dissertation Abstracts International 33A: 1022; September 1972.

Isaac, Jacob. The Use of CAI in the Teaching of Logarithms: An Evaluative and Experimental Study. University of Alberta, 1972. 161 p.

Jerman, M. and Rees, R. Predicting the Relative Difficulty of Verbal Arithmetic Problems. Educational Studies in Mathematics 4: 306-323; April 1972.

Johnson, David C. Preliminary Reports I and II: Computer Assisted Mathematics Project. University of Minnesota, 1966.

Johnson, Randall Erland. The Effect of Activity Oriented Lessons on the Achievement and Attitudes of Seventh Grade Students in Mathematics. (University of Minnesota, 1970.) Dissertation Abstracts International 32A: 305; July 1971.

Jurick, Robert Rudolph. Computer-Oriented Instructional System for Teaching Analytic Geometry. (The Ohio State University, 1972.) Dissertation Abstracts International 33B: 1667-1668: October 1972.

Kanes, Lelage C. A Comparison of Two Teaching Strategies Used to Present a Unit in Elementary Mathematics Using Computer-Assisted Instruction. (University of Pennsylvania, 1971.) Dissertation Abstracts International 32A: 1981; October 1971.

Katz, Saul M. A Comparison of the Effects of Two Computer Augmented Methods of Instruction with Traditional Methods upon Achievement of Algebra Two Students in a Comprehensive High School. (Temple University, 1971.) Dissertation Abstracts International 32A: 1188-1189; September 1971.

Kieren, Thomas Ervin. The Computer as a Teaching Aid for Eleventh Grade Mathematics: A Comparison Study. University of Minnesota, 1968. 133 p. (See also Dissertation Abstracts 29A: 3526-3527; April 1969.)

King, Donald Thomas. An Instructional System for the Low-Achiever in Mathematics: A Formative Study. (The University of Wisconsin, 1972.) Dissertation Abstracts International 32A: 6743; June 1972.

Klein, Paul Alfred. Toward a Theory of Sequencing: Study 1-1: An Exploration of Selected Relationships Among the Enactive, Iconic and Symbilic Modes of Representation. Journal for Research in Mathematics Education 4: 94-103; March 1973.

Klein, Paul A. Toward a Theory of Sequencing: Study 1-1: An Exploration of Selected Relationships Among the Enactive, Iconic and Symbolic Modes of Representation. The Pennsylvania State University, 1970. 150 p. (See also Dissertation Abstracts International 31A: 4624-4625; March 1971.)

Klement, Jerome Joseph. The Effects of Three Reinforcement Schedules and Two Media in Presenting Computer Assisted Instruction to Under-Educated Adults. (North Carolina State University at Raleigh, 1971.) Dissertation Abstracts International 32A: 1830-1831; October 1971.

Knodel, Melvin Leonard. The Effect of Curriculum Sequences Upon Student Achievement in Selected Computer Languages. (The University of North Dakota, 1970.) Dissertation Abstracts International 32A: 6743-6744; June 1972.

Kriewall, Thomas Edward. Applications of Information Theory and Acceptance Sampling Principles to the Management of Mathematics Instruction. (The University of Wisconsin, 1969.) Dissertation Abstracts International 30A: 5344; June 1970.

Lee, Kwi-Yoon. A Study and Analysis of the Effectiveness of Computer Assisted Reporting of Fifth Grade Pupils' Mathematical Progress as Perceived by Parents and Pupils. (The University of Iowa, 1972.) Dissertation Abstracts International 33A: 1368; October 1972.

Love, William Pegram. Individual Versus Paired Learning of an Abstract Algebra Presented by Computer Assisted Instruction. (The Florida State University, 1969.) Dissertation Abstracts International 31A: 248; July 1970.

Luskin, Bernard Jay. An Identification and Examination of Obstacles to the Development of Computer Assisted Instruction. (University of California, Los Angeles, 1970.) Dissertation Abstracts International 31B: 3980-3981; January 1971.

Lynch, Robert Michael. A Cost Effectiveness Analysis of Computer Assisted Instruction. (University of Northern Colorado, 1971.) Dissertation Abstracts International 32B: 839: August 1971.

Melaragno, Ralph James. A Comparison of Two Methods of Adapting Self-Instructional Materials to Individual Differences Among Learners. (University of Southern California, 1966.) Dissertation Abstracts 27B: 3273-3274; March 1967.

Morgan, Richard Thomas. The Role of the Digital Computer in a General Education Course in Mathematics. (Columbia University, 1968.) Dissertation Abstracts International 30A: 71-72; July 1969.

Oettinger, A. with Marks, Sema. Run Computer, Run: The Mythology of Educational Innovation: An Essay. Cambridge, Massachusetts: Harvard University Press, 1969. 302 p.

O'Neil, Harold F., Jr. Effects of Stress on State Anxiety and Performance in Computer-Assisted Learning. (The Florida State University, 1969.) Dissertation Abstracts International 31B: 1568; September 1970.

Ostheller, Karl Olney. The Feasibility of Using Computer-Assisted Instruction to Teach Mathematics in the Senior High School. (Washington State University, 1970.) Dissertation Abstracts International 31A: 4042; February 1971.

Pack, Elbert Chandler. The Effect of Mode of Computer Operation on Learning a Computer Language and on Problem Solving Efficiency of College Bound High School Students. (University of California, Los Angeles, 1970.) Dissertation Abstracts International 31A: 6477; June 1971.

Papert, S. and Soloman, C. Twenty Things to Do With A Computer. Educational Technology 12: 9-18; April 1972.

Paquette, G. A. Toward a Theory of Sequencing: Study 1-3: An In-Depth Exploration of the Role of Iconic Representations in the Study of Congruence of Triangles. Journal for Research in Mathematics Education 4: 105-110; March 1973.

Parkus, L. CAI in Elementary and Secondary Education: The State of the Art. To Improve Learning, Volume I. S. G. Tickton, Editor, R. R. Bowker Co., N.Y., 1970. pp. 323-338.

Pride, Bonnie Loraine. A Critical Analysis of Computer Utilization by Mathematics Departments in Selected Small Colleges. (George Peabody College for Teachers. 1972.) Dissertation Abstracts International 33B: 1674-1675; October 1972.

Richard, Howard Marks Simon. New Careers Mathematics: The Effect Upon Achievement in Mathematics of Supplementing a Concept Center Course for Adults with Experiences in Computer Utilization. (The Ohio State University. 1970.) Dissertation Abstracts International 31B: 4209-4210; January 1971.

Riedesel, C. Alan and Suydam, Marilyn N. Computer-Assisted Instruction: Implications for Teacher Education. The Arithmetic Teacher 14: 24-29; January 1967.

Ronan, Franklin Delano. Study of the Effectiveness of a Computer When Used as a Teaching and Learning Tool in High School Mathematics. (The University of Michigan, 1971.) Dissertation Abstracts International 32A: 1264-1265; September 1971.

Sawada, Daiya. Piaget and Pedagogy: Fundamental Relationships. The Arithmetic Teacher 19: 293-298; April 1972.

Sawada, Daiya. Toward a Theory of Sequencing: Study 3-1: An Assessment of a Selected Set of Piagetian-Derived Operators for the Generation of Effective Learning Hierarchies. Journal for Research in Mathematics Education 4: 118-121; March 1973.

Schmidt, H. E. The Use of the Computer as a Unique Teaching Tool for Introductory Calculus. Final Report. Ferguson, Missouri: Florissant Valley Community College, June 1970. ERIC: ED 042 655. 37 p.

Schoen, Harold Leo. A Comparison of Four Types of Feedback to Student Responses in a CAI Unit Designed to Teach the Concept of Function to Pre-Calculus College Students. (The Ohio State University, 1971.) Dissertation Abstracts International 32A: 2508; November 1971. (See also ERIC: ED 061 081.)

Schurdak, John L. An Approach to the Use of Computers in the Instructional Process and an Evaluation. American Educational Research Journal 4: 59-73; January 1967.

Shaw, Carl Neil. Effects of Three Instructional Strategies on Achievement in a Remedial Arithmetic Program. (The Florida State University, 1968.) Dissertation Abstracts 29A: 1479-1480; November 1968.

Skelton, John E. Time-Sharing Versus Batch Processing in Teaching Beginning Computer Programming: An Experiment. Part One of a Two-Part Article. AEDS Journal 5: 91-97; March 1972. ERIC: EJ 055 560.

Smith, David A. A Calculus with Computer Experiment. Educational Studies in Mathematics 3: 1-11; 1971.

S.R.A. 1968-69 Arithmetic Proficiency Program, Field Test Summary. Chicago: Science Research Associates, 1970. 4 p.

Suppes, Patrick. Computer-Based Mathematics Instruction. Stanford: Institute for Mathematical Studies in the Social Sciences, Stanford University, September 1964.

Suppes, P. Some Theoretical Models for Mathematics Learning. Journal of Research and Development in Education 1: 5-22; Fall 1967.

Suppes, Patrick; Jerman, Max; and Dow, Brian. Computer-Assisted Instruction: Stanford's 1965-66 Arithmetic Program. New York: Academic Press, 1968.

Suppes, Patrick; Jerman, Max; and Groen, Guy. Arithmetic Drills and Review on a Computer-Based Teletype. The Arithmetic Teacher 13: 303-309; April 1966. (See also: ERIC: ED 014 215. 17 p.)

Suppes, Patrick; Loftus, Elizabeth F.; and Jerman, Max. Problem Solving on a Computer-Based Teletype. Stanford: Institute for Mathematical Studies in the Social Sciences, Stanford University, March 1969. ERIC: ED 029 795. 26 p.

Suppes, Patrick and Morningstar, Mona. Computer-Assisted Instruction: The 1966-67 Stanford Arithmetic Program. New York: Academic Press, 1969.

Walbesser, Henry H. Behavioral Objectives: A Cause Celebre. The Arithmetic Teacher 19: 418-440; October 1972.

Ward, R. G., and Ballew, L. N. The Experimental Use of a Computer to Teach Basic Set Theory. Journal of Educational Data Processing 9: 1-8; 1972.

Washburn, Robert Miles. CEMP-A Computer Enriched Mathematics Program. (Cornell University, 1969.) Dissertation Abstracts International 30A: 5179; June 1970.

Westrom, M. L. Teacher Authored Instruction Manager. In Canadian Symposium on Educational Technology, Calgary, May 1972.

Whitcomb, Barry Michael. The Effects of Differentially Reinforcing Stimuli on the Learning of Arithmetical Concepts via Computer-Assisted Instruction. (University of Maryland, 1972.) Dissertation Abstracts International 33A: 5[illegible]9; August 1972.

SUPPLEMENTARY REFERENCES

Ackermann, Arthur Frank, Jr. Toward a Programming Language for Writing and Checking Mathematical Discourses. (University of North Carolina at Chapel Hill, 1972.) Dissertation Abstracts International 33B: 1495-1496; October 1972.

Barry, Linda. Musical Math and Binary Bach. AEDS Monitor 11: December 1972.

Bitter, Gary. Computer-Oriented Calculus. Journal of Educational Data Processing 7: 193-195; March 1970. ERIC: EJ 027 020.

Bowers, Robert G rth. Toward a Theory of Sequencing: Study 1-4: An Exploration of the Effect of Congruent Triangle-Pair Configuration Variation on Achievement of Selected Classes of Instructional Objectives in Plane Geometry. (The Pennsylvania State University, 1971.) Dissertation Abstracts International 32A: 6181-6182; May 1972.

Dennis, John Richard. Teaching Selected Geometry Topics Via A Computer System. (University of Illinois, 1968.) Dissertation Abstracts 29A: 2145; January 1969.

Farris, Dan Curry. Toward a Theory of Sequencing: Study 1-2: An Exploration of Selected Relationships Among the Enactive, Iconic and Symbolic Modes of Representation. (The Pennsylvania State University, 1970.) Dissertation Abstracts International 31A: 4618; March 1971.

Feurzig, W. and Lukas, S. LOGO--A Programming Language for Teaching Mathematics. Educational Technology 12: 39-45; March 1972.

Gaines, B. R. The Learning of Pre-Motor Skills by Man and Machines and Its Relationship to Training. Instructional Science 1: 263-309; October 1972.

Graham, Ray Logan. An Investigation of the Effects of Computer upon Elementary Analysis. (New Mexico State University, 1968.) Dissertation Abstracts 29B: 1431; October 1968.

Heimer, Ralph T. Conditions of Learning in Mathematics: Sequence Theory Development. Review of Educational Research 38: 493-508; October 1969.

Hirschbuhl, John Joseph. Toward A Theory of Sequencing: Study 1-5: An Exploration of Selected Transitivity and Conjunctive Relationships Among the Enactive, Iconic and Symbolic Modes of Representation. (The Pennsylvania State University, 1971.) Dissertation Abstracts International 32A: 6202; May 1972.

Holland, J. G. and Daron, J. Teaching Classification by Computer. Educational Technology 12: 58-60; December 1972.

Hopkins, Layne Victor. Toward A Theory of Sequencing: Study 3-2: An Exploration of Transitivities Formulated From a Set of Piagetian-Derived Operations and Their Implications in Traversing Learning Hierarchies. (The Pennsylvania State University, 1971.) Dissertation Abstracts International 32A: 6203: May 1972.

Jerman, Max. Some Strategies for Solving Simple Multiplication Combinations. Journal for Research in Mathematics Education 1: 95-128; March 1970. ERIC: EJ 038 736.

Loftus, Elizabeth Jane Fishman. An Analysis of the Structural Variables that Determine Problem-Solving Difficulty on a Computer-Based Teletype. (Stanford University, 1970.) Dissertation Abstracts International 31A: 5853: May 1971.

McClain, Donald Henry. Development of a Computer-Assisted Instruction Unit in Probability. (Iowa State University, 1970.) Dissertation Abstracts International 31B: 5310; March 1971.

Miller, George Eugene. A Computer-Assisted Experiment on the Process of Learning and Problem Solving Under the Condition of Goal-Direction. (University of Maryland, 1969.) Dissertation Abstracts International 31A: 1081-1082; September 1970.

Moloney, James Michael. An Investigation of College Student Performance of a Logic Curriculum in a Computer-Assisted Instruction Setting. (Stanford University, 1972.) Dissertation Abstracts International 32A: 6851; June 1972. Also Stanford: Institute for Mathematical Studies in the Social Sciences, Stanford University, January 1972. ERIC: ED 062 780.

O'Neil, H. F. Effects of State Anxiety and Task Difficulty on Computer-Assisted Learning. Journal of Educational Psychology 60: 343-350: October 1969.

Rosenbaum, Sema Joy Marks. A Course in Computer Simulation for High School Students. (Harvard University, 1970.) Dissertation Abstracts International 31A: 5676; May 1971.

Rudolph, Eleanore L. A Survey of Data Processing and Computer Use in Instruction in Illinois Secondary Schools, (Northern Illinois University, 1972.) Dissertation Abstracts International 33A: 505-506; August 1972.

Spencer, D. The Computer Goes to School. AEDS Journal 6: 3-30: Winter 1972.

Wexler, Jonathan David. A Generative, Remedial, and Query System for Teaching by Computer. (University of Wisconsin, 1970.) Dissertation Abstracts International 31B: 2623-2624: November 1970.

Young, James Heyward, Jr. The Use of a Computer-Based Resource Guide to Pre-Plan a Unit of Instruction and to Develop Student Attitudes Toward Mathematics. (State University of New York at Buffalo, 1970.) Dissertation Abstracts International 31A: 1790: October 1970.